A generalization of Clements' method for non-normal Pearsonian processes with asymmetric tolerances

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Abstract *For non-normal Pearsonian processes, Clements proposed a method for calculating estimators of the two basic process capability indices C_p and C_{pk}. Pearn and Kotz applied Clements' method to obtain estimators for the other two more advanced process capability indices C_{pm} and C_{pmk}. Their considerations on those indices, however, are restricted to processes with symmetric tolerances. Recently, Pearn and Chen proposed a generalization of the index C_{pk} to handle cases with asymmetric tolerances. The generalization takes into account the asymmetry of the manufacturing specifications, which is shown to be superior to the other existing methods. In this paper, we apply this approach and consider a generalization of Clements' method for non-normal Pearsonian processes where the manufacturing tolerances are asymmetric. Comparisons between the original Clements' method and the proposed generalization are provided. The results indicate that the generalization is more accurate than the original Clements' method in measuring process capability.*

1. Introduction

Process capability indices (PCIs) have been widely used in the manufacturing industry, to provide numerical measures on whether a process is capable of producing items meeting the quality requirement preset in the factory. Numerous capability indices have been proposed to measure process potential and performance. Examples include the two most commonly used indices C_p and C_{pk} discussed in Kane (1986), and the two more-advanced indices C_{pm} and C_{pmk} developed by Chan *et al.* (1988), and Pearn *et al.* (1992). There are many other indices, but they can be viewed as modifications of these four basic capability indices. The indices C_p, C_{pk}, C_{pm}, and C_{pmk} can be defined as the following:

$$C_p = \frac{\mathrm{USL} - \mathrm{LSL}}{6\sigma},$$

$$C_{pk} = \min\left\{\frac{\mathrm{USL} - \mu}{3\sigma}, \frac{\mu - \mathrm{LSL}}{3\sigma}\right\},$$

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$$C_{pm} = \frac{\text{USL} - \text{LSL}}{6\sqrt{\sigma^2 + (\mu - T)^2}},$$

$$C_{pmk} = \min\left\{\frac{\text{USL} - \mu}{3\sqrt{\sigma^2 + (\mu - T)^2}}, \frac{\mu - \text{LSL}}{3\sqrt{\sigma^2 + (\mu - T)^2}}\right\},$$

where USL is the upper specification limit, LSL is the lower specification limits, μ is the process mean, σ is the process standard deviation, and T is the target value. While C_p measures the overall process variation relative to the specification tolerance, C_{pk} takes into account the proximity of the process mean to the center of the specification tolerance as well as the process variation in the assessment of process performance, which is essentially a measure of the process yield. In fact, the process yield can be calculated as $2\Phi\ (3C_{pk}) - 1 < \text{Yield} < \Phi(3C_{pk})$ if the process follows the normal distribution, where $\Phi\ (\cdot)$ is the cumulative function for the standard normal distribution. On the other hand, C_{pm} and C_{pmk} take into account the proximity of the process mean to the target (rather than the center), which are more sensitive to the process departure than C_p and C_{pk}.

Discussions and analysis of the four basic indices on point estimation, the construction of confidence intervals, and the testing hypothesis on process capability for decision-making purposes, have been the focus of many statistician and quality researchers including Chan *et al.* (1988), Chou *et al.* (1989), Pearn *et al.* (1992), Kushler and Hurley (1992), Franklin and Wasserman (1992), Vännman (1994), Pearn and Chen (1995; 1998), and many others. Most of the investigations, however, depend heavily on the assumption of normal variability. If the underlying distributions are non-normal, then the capability calculations are highly unreliable since the conventional estimator S^2 of σ^2 is sensitive to departures from normality, and estimators of those indices are calculated using S^2. In fact, Gunter (1989) demonstrated the strong impact this has on the sampling distribution of the natural estimator of C_{pk}. Therefore, the natural (conventional) estimators of those basic indices are inappropriate for non-normal processes.

For non-normal distributions, Clements (1989) and Pearn and Kotz (1994) considered a method for calculating estimators of those indices assuming that the process follows the Pearsonian distribution. But their considerations are restricted to processes with symmetric tolerances. In this paper, we consider a generalization of their method to handle cases with asymmetric tolerances. The generalization takes into account the asymmetry of the tolerances, which is more sensitive than the original Clements' and the modified Clements' methods in detecting process shift. The results also show that the proposed generalization is more accurate than the original Clements' and the modified Clements' methods in measuring process capability. An example on the MOSFET manufacturing process, illustrating how we may apply the proposed generalization, is provided.

2. Clements' method

For non-normal Pearsonian distributions (which include a rich class of populations with non-normal characteristics), Clements (1989) proposed a method for calculating estimators of C_p and C_{pk}. Pearn and Kotz (1994) applied Clements' method to obtain estimators for the other two basic indices, C_{pm} and C_{pmk}. Those estimators have been defined as:

$$\hat{C}_p = \frac{\text{USL} - \text{LSL}}{U_p - L_p},$$

$$\hat{C}_{pk} = \min\left\{\frac{\text{USL} - M}{U_p - M}, \frac{M - \text{LSL}}{M - L_p}\right\},$$

$$\hat{C}_{pm} = \frac{\text{USL} - \text{LSL}}{6\sqrt{[\frac{U_p - L_p}{6}]^2 + (M - T)^2}},$$

$$\hat{C}_{pmk} = \min\left\{\frac{\text{USL} - M}{3\sqrt{[\frac{U_p - M}{3}]^2 + (M - T)^2}}, \frac{M - \text{LSL}}{3\sqrt{\left[\frac{M - L_p}{3}\right]^2 + (M - T)^2}}\right\},$$

where U_p and L_p are the 99.865 and the 0.135 percentiles determined from Gruska *et al.* table (1989) for the particular values of mean, variance, skewness, and kurtosis calculated from the sample data. For the indices C_p and C_{pm}, Clements' estimators are obtained by replacing the 6σ by $U_p - L_p$.

For the indices C_{pk} and C_{pmk}, Clements' estimators are obtained by replacing the two 3σ by $U_p - M$ and $M - L_p$ respectively for the right-hand and left-hand sides. We note that the process mean μ is also replaced by the process median M since the process median is a more robust measure of central tendency than the process mean, particularly for skewed distributions with long tails. The idea behind such replacements is to mimic the property of the normal distribution for which the tail probability outside the $\pm 3\sigma$ limits from μ is 0.27 percent thus assuring that if the calculated value of $C_p = 1$ (assuming the process is well-centered), then the probability that process is outside the specification limits (LSL, USL) will be negligibly small.

To improve the accuracy of Clements' method in measuring the process capability, Pearn and Chen (1995) considered the following modification which replaces the σ by $(U_p - L_p)/6$ for all cases regardless of whether it is on the right-hand side or left-hand side. Thus, Clements' estimators become:

$$\hat{C}'_p = \frac{\text{USL} - \text{LSL}}{U_p - L_p},$$

$$\hat{C}'_{pk} = \min\left\{\frac{\text{USL} - M}{[U_p - L_p]/2}, \frac{M - \text{LSL}}{[U_p - L_p]/2}\right\},$$

$$\hat{C}'_{pm} = \frac{\text{USL} - \text{LSL}}{6\sqrt{[\frac{U_p - L_p}{6}]^2 + (M - T)^2}},$$

$$\hat{C}'_{pmk} = \min\left\{\frac{\text{USL} - M}{3\sqrt{[\frac{U_p - L_p}{6}]^2 + (M - T)^2}}, \frac{M - \text{LSL}}{3\sqrt{[\frac{U_p - L_p}{6}]^2 + (M - T)^2}}\right\},$$

To illustrate how the modified estimators outperform the original Clements' estimators, Pearn and Chen (1995) considered an example of three processes with one on-target and the other two off-target. While Clements' estimators show little sensitivity to the departure of the process median from the target value, the modified estimators clearly differentiate the on-target process from the other two (severely) off-target processes.

3. A generalization for asymmetric tolerances

Although cases with symmetric tolerances (USL – T = T – LSL) are quite common in practical situations, there are other situations where the tolerances are asymmetric (USL – $T \neq T$ – LSL). For asymmetric tolerances, Kane (1986) considered a method which shifts one of the two specification limits so that the shifted specification limits are symmetric to the target value T. The method transforms the original specifications (LSL, T, LSL) into (T – min{USL – T, T – LSL}, T, T + min{USL – T, T – LSL}). Kushler and Hurley (1992) considered a different method which shifts both of the two specification limits so that the shifted specification limits are symmetric to the target value T. The method transforms the specifications (LSL, T, LSL) into (T – [USL – LSL]/2, T, T + [USL – LSL]/2). These two methods are straightforward to apply. Unfortunately, these two methods can severely understate or overstate process capability (Pearn and Chen, 1998), thus reflecting process performance inaccurately. Consequently, they are inappropriate for processes with asymmetric tolerances.

To overcome the problem, Vännman (1997) considered an alternative method to handle cases with asymmetric tolerances. The method modifies the basic indices by adding a new term $|\mu - T|$ in the numerator of the definitions. Pearn *et al.* (1998) investigated Vännman's method, and pointed out that this method can severely understate or overstate process capability. Therefore, Vänman's method is not appropriate for processes with asymmetric tolerances. Recently, Pearn and Chen (1998) considered a new method, and obtained a generalization of C_{pk} for asymmetric tolerances. The method takes into account the asymmetry of the corresponding loss function, which is shown to be superior to the other existing methods. In this paper, we apply this method and consider a generalization of Clements' method for non-normal Pearsonian processes where the tolerances are asymmetric. Comparisons between the original Clements' method and the proposed generalization are provided. The generalizations are defined as:

$$\hat{C}''_p = \frac{2 \times d^*}{U_p - L_p},$$

$$\hat{C}''_{pk} \min\left\{\frac{\mathrm{USL} - M}{[U_p - L_p]/2} \times \frac{d^*}{d_u}, \frac{M - \mathrm{LSL}}{[U_p - L_p]/2} \times \frac{d^*}{d_l}\right\},$$

$$\hat{C}''_{pm} = \frac{2 \times d^*}{6\sqrt{[\frac{U_p - L_p}{6}]^2 + a^2}},$$

$$\hat{C}''_{pmk} = \min\left\{\frac{\mathrm{USL} - M}{3\sqrt{\left[\frac{U_p - L_p}{6}\right]^2 + a^2}} \times \frac{d^*}{d_u}, \frac{M - \mathrm{LSL}}{3\sqrt{\left[\frac{U_p - L_p}{6}\right]^2 + a^2}} \times \frac{d^*}{d_l}\right\},$$

where $d^* = \min\{du, d_l\}$, $d_u = \mathrm{USL} - T$, $d_l = T - \mathrm{LSL}$, $d = (\mathrm{USL} - \mathrm{LSL})/2$, and $a = \max\{d(M - T)/d_u, d(T - M)/d_l\}$. Clearly, if $T = m$ (symmetric case) then $a = |M - T|$ and the new estimators $\hat{C}''_p$, $\hat{C}''_{pk}$, $\hat{C}''_{pm}$, $\hat{C}''_{pmk}$ reduce to the modified estimators $\hat{C}'_p$, $\hat{C}'_{pk}$, $\hat{C}'_{pm}$ and $\hat{C}'_{pmk}$ considered by Pearn and Chen (1995). The factors d^* and a ensure that the new estimators obtain their maximal value at $M = T$ regardless of whether the tolerances are symmetric or asymmetric.

For processes with asymmetric tolerances, the corresponding loss function is also asymmetric to the target value T. Figure 1 displays a typical loss function for processes with asymmetric tolerances. The loss function depicted in Figure 1 (assumed quadratic, a popular one considered in many applications) is defined in the following with value setting to 1 for x falling outside the manufacturing specification limits, LSL and USL.

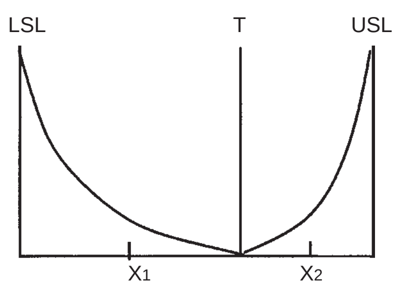

Figure 1. An asymmetric loss function

$$L(x) = \left(\frac{T - x}{T - \mathrm{LSL}}\right)^2, \ \mathrm{LSL} \ < x \leq T,$$
$$= \left(\frac{x - T}{\mathrm{USL} - T}\right)^2, \ T \leq x < \mathrm{USL},$$
$$= 1, \ \text{otherwise}$$

We note that for the mid-point of the left-hand side tolerance, $x_1 = (T + \mathrm{LSL})/2$ and the mid-point of the right-hand side tolerance, $x_2 = (T + \mathrm{USL})/2$, the corresponding loss can be calculated as:

$$L(x_1) = \left(\frac{T - x_1}{T - \mathrm{LSL}}\right)^2$$
$$= \left[\frac{T - \frac{\mathrm{LSL}+T}{2}}{T - \mathrm{LSL}}\right]^2 = \frac{1}{4},$$

$$L(x_2) = \left(\frac{x_2 - T}{\mathrm{USL} - T}\right)^2$$
$$= \left[\frac{\frac{\mathrm{USL}+T}{2} - T}{\mathrm{USL} - T}\right] = \frac{1}{4}.$$

Obviously, the two points x_1 and x_2 have the same departure ratio (relative departure) $k = (T - x_1)/d_l = (x_2 - T)/d_u = 1/2$. Checking the process loss at x_1 and x_2, we have $L(x_1) = L(x_2) = 1/4$.

4. Performance comparisons

To illustrate how the new generalizations incorporate the asymmetric loss function, we consider the following example with asymmetric tolerance (LSL, T, USL) = (15, 45, 60), with fixed process variations $U_p - L_p = 0.8$d, $U_p - M =$ 0.5d, and $M - L_p = 0.3$d, where $15 \leq M \leq 60$. Tables I, II and III display the values of the original Clements' estimators, $\hat{C}_p$, $\hat{C}_{pk}$, $\hat{C}_{pm}$, $\hat{C}_{pmk}$, the modified Clements' estimators, $\hat{C}'_p$, $\hat{C}'_{pk}$, $\hat{C}'_{pm}$, $\hat{C}'_{pmk}$ and the new estimators, $\hat{C}''_p$, $\hat{C}''_{pk}$, $\hat{C}''_{pm}$ and $\hat{C}''_{pmk}$ for those processes. We note that the new estimators $\hat{C}''_p$, $\hat{C}''_{pk}$, $\hat{C}''_{pm}$ and $\hat{C}''_{pmk}$ obtain their maximal values at the target value $T = 45$.

Table IV summarizes the values of the new estimators obtained for processes with equal departure ratio, thus satisfying the condition $(M_\mathrm{A} - T)/D_u = (T - M_\mathrm{B})/D_l)$. For example, consider processes A and B with $M_\mathrm{A} = 50$ and $M_\mathrm{B} = 35$. It is easy to verify that for processes A and B $(M_\mathrm{A} - T)/D_u = (50 - 45)/15 = 1/3$, and $(T - M_\mathrm{B})/D_l = (45 - 35)/30 = 1/3$, thus satisfying $(M_\mathrm{A} - T)/D_u = (T - M_\mathrm{B})/D_l$. Process loss for A and B would be the same in this case. Checking Table IV, we have the same index values for both processes A and B.

M	$\hat{C}_p$	$\hat{C}_{pk}$	$\hat{C}_{pm}$	$\hat{C}_{pmk}$
15	2.500	0.000	0.249	0.000
16	2.500	0.148	0.257	0.011
17	2.500	0.296	0.266	0.024
18	2.500	0.444	0.276	0.037
19	2.500	0.593	0.287	0.051
20	2.500	0.741	0.298	0.066
21	2.500	0.889	0.310	0.083
22	2.500	1.037	0.323	0.101
23	2.500	1.185	0.338	0.121
24	2.500	1.333	0.354	0.142
25	2.500	1.481	0.371	0.166
26	2.500	1.630	0.390	0.192
27	2.500	1.778	0.411	0.221
28	2.500	1.926	0.434	0.253
29	2.500	2.074	0.461	0.289
30	2.500	2.222	0.490	0.330
31	2.500	2.370	0.524	0.376
32	2.500	2.489	0.562	0.430
33	2.500	2.400	0.606	0.491
34	2.500	2.311	0.658	0.564
35	2.500	2.222	0.718	0.650
36	2.500	2.133	0.791	0.755
37	2.500	2.044	0.878	0.868
38	2.500	1.956	0.985	0.923
39	2.500	1.867	1.118	0.989
40	2.500	1.778	1.286	1.067
41	2.500	1.689	1.500	1.155
42	2.500	1.600	1.768	1.249
43	2.500	1.511	2.080	1.333
44	2.500	1.422	2.372	1.374
45	2.500	1.333	2.500	1.333
46	2.500	1.244	2.372	1.202
47	2.500	1.156	2.080	1.020
48	2.500	1.067	1.768	0.833
49	2.500	0.978	1.500	0.669
50	2.500	0.889	1.286	0.533
51	2.500	0.800	1.118	0.424
52	2.500	0.711	0.985	0.336
53	2.500	0.622	0.878	0.264
54	2.500	0.533	0.791	0.205
55	2.500	0.444	0.718	0.156
56	2.500	0.356	0.658	0.115
57	2.500	0.267	0.606	0.080
58	2.500	0.178	0.562	0.049
59	2.500	0.089	0.524	0.023
60	2.500	0.000	0.490	0.000

Table I. Values of $\hat{C}_p, \hat{C}_{pk}, \hat{C}_{pm}$, and $\hat{C}_{pmk}$ for processes with $T = 45$, $U_p - L_p = 0.8d$, $U_p - M = 0.5d$, $M - L_p = 0.3d$ and $15 \leq M \leq 60$

To further show how the new estimators outperform the other estimators in detecting the process shifting, we consider the following example with the on-target process A, and three shifted processes A_1, A_2, and A_3, where the

M	$\hat{C}'_p$	$\hat{C}'_{pk}$	$\hat{C}'_{pm}$	$\hat{C}'_{pmk}$
15	2.500	0.000	0.249	0.000
16	2.500	0.111	0.257	0.011
17	2.500	0.222	0.266	0.024
18	2.500	0.333	0.276	0.037
19	2.500	0.444	0.287	0.051
20	2.500	0.556	0.298	0.066
21	2.500	0.667	0.310	0.083
22	2.500	0.778	0.323	0.101
23	2.500	0.889	0.338	0.120
24	2.500	1.000	0.354	0.141
25	2.500	1.111	0.371	0.165
26	2.500	1.222	0.390	0.191
27	2.500	1.333	0.411	0.219
28	2.500	1.444	0.434	0.251
29	2.500	1.556	0.461	0.287
30	2.500	1.667	0.490	0.327
31	2.500	1.778	0.524	0.372
32	2.500	1.889	0.562	0.425
33	2.500	2.000	0.606	0.485
34	2.500	2.111	0.658	0.555
35	2.500	2.222	0.718	0.639
36	2.500	2.333	0.791	0.738
37	2.500	2.444	0.878	0.858
38	2.500	2.444	0.985	0.963
39	2.500	2.333	1.118	1.043
40	2.500	2.222	1.286	1.143
41	2.500	2.111	1.500	1.267
42	2.500	2.000	1.768	1.414
43	2.500	1.889	2.080	1.572
44	2.500	1.778	2.372	1.687
45	2.500	1.667	2.500	1.667
46	2.500	1.556	2.372	1.476
47	2.500	1.444	2.080	1.202
48	2.500	1.333	1.768	0.943
49	2.500	1.222	1.500	0.733
50	2.500	1.111	1.286	0.572
51	2.500	1.000	1.118	0.447
52	2.500	0.889	0.985	0.350
53	2.500	0.778	0.878	0.273
54	2.500	0.667	0.791	0.211
55	2.500	0.556	0.658	0.160
56	2.500	0.444	0.658	0.117
57	2.500	0.333	0.606	0.081
58	2.500	0.222	0.562	0.050
59	2.500	0.111	0.524	0.023
60	2.500	0.000	0.490	0.000

Table II.
Values of $\hat{C}'_p$, $\hat{C}'_{pk}$, $\hat{C}'_{pm}$, and $\hat{C}'_{pmk}$ for processes with $T = 45$, $U_p - L_p = 0.8d$, $U_p - M = 0.5d$, $M - L_p = 0.3d$ and $15 \leq M \leq 60$

manufacturing tolerance (LSL, T, USL) = (20, 50, 60). Figure 2(a) displays the histogram of the data from the on-target process A. Figure 2(b) displays the

M	$\hat{C}''_p$	$\hat{C}''_{pk}$	$\hat{C}''_{pm}$	$\hat{C}''_{pmk}$
15	1.667	0.000	0.220	0.000
16	1.667	0.056	0.228	0.008
17	1.667	0.111	0.236	0.016
18	1.667	0.167	0.244	0.024
19	1.667	0.222	0.253	0.034
20	1.667	0.278	0.263	0.044
21	1.667	0.333	0.274	0.055
22	1.667	0.389	0.286	0.067
23	1.667	0.444	0.298	0.080
24	1.667	0.500	0.312	0.094
25	1.667	0.556	0.327	0.109
26	1.667	0.611	0.343	0.126
27	1.667	0.667	0.362	0.145
28	1.667	0.722	0.382	0.165
29	1.667	0.778	0.404	0.189
30	1.667	0.833	0.429	0.215
31	1.667	0.889	0.458	0.244
32	1.667	0.944	0.490	0.278
33	1.667	1.000	0.527	0.316
34	1.667	1.056	0.570	0.361
35	1.667	1.111	0.619	0.413
36	1.667	1.167	0.677	0.474
37	1.667	1.222	0.745	0.547
38	1.667	1.278	0.827	0.634
39	1.667	1.333	0.925	0.740
40	1.667	1.389	1.041	0.868
41	1.667	1.444	1.179	1.021
42	1.667	1.500	1.333	1.200
43	1.667	1.556	1.491	1.391
44	1.667	1.611	1.617	1.563
45	1.667	1.667	1.667	1.667
46	1.667	1.556	1.491	1.391
47	1.667	1.444	1.179	1.021
48	1.667	1.333	0.925	0.740
49	1.667	1.222	0.745	0.547
50	1.667	1.111	0.619	0.413
51	1.667	1.000	0.527	0.316
52	1.667	0.889	0.458	0.244
53	1.667	0.778	0.404	0.189
54	1.667	0.667	0.362	0.145
55	1.667	0.556	0.327	0.109
56	1.667	0.444	0.298	0.080
57	1.667	0.333	0.274	0.055
58	1.667	0.222	0.253	0.034
59	1.667	0.111	0.236	0.016
60	1.667	0.000	0.220	0.000

Table III. Values of $\hat{C}''_p, \hat{C}''_{pk}, \hat{C}''_{pm}$, and $\hat{C}''_{pmk}$ for processes with $T = 45$, $U_p - L_p = 0.8d$, $U_p - M = 0.5d$, $M - L_p = 0.3d$ and $15 \leq M \leq 60$

histogram of the data from the shifted process A_1. Figure 2(c) displays the histogram of the data from the shifted process A_2. Figure 2(d) displays the histogram of the data from the shifted process A_3.

M	$\hat{C}''_p$	$\hat{C}''_{pk}$	$\hat{C}''_{pm}$	$\hat{C}''_{pmk}$
15	1.667	0.000	0.220	0.000
60	1.667	0.000	0.220	0.000
17	1.667	0.111	0.236	0.016
59	1.667	0.111	0.236	0.016
19	1.667	0.222	0.253	0.034
58	1.667	0.222	0.253	0.034
21	1.667	0.333	0.274	0.055
57	1.667	0.333	0.274	0.055
23	1.667	0.444	0.298	0.080
56	1.667	0.444	0.298	0.080
25	1.667	0.556	0.327	0.109
55	1.667	0.556	0.327	0.109
27	1.667	0.667	0.362	0.145
54	1.667	0.667	0.362	0.145
29	1.667	0.778	0.404	0.189
53	1.667	0.778	0.404	0.189
31	1.667	0.889	0.458	0.244
52	1.667	0.889	0.458	0.244
33	1.667	1.000	0.527	0.316
51	1.667	1.000	0.527	0.316
35	1.667	1.111	0.619	0.413
50	1.667	1.111	0.619	0.413
37	1.667	1.222	0.745	0.547
49	1.667	1.222	0.745	0.547
39	1.667	1.333	0.925	0.740
48	1.667	1.333	0.925	0.740
41	1.667	1.444	1.179	1.021
47	1.667	1.444	1.179	1.021
43	1.667	1.556	1.491	1.391
46	1.667	1.556	1.491	1.391

Table IV.
The values of the new estimators for processes with $(\mu_A - T)/D_\mu = (T_{\mu B})/D_l$

Table V displays the characteristics of the data for processes A, A_1, A_2, A_3, the values of Clements' estimators, the modified estimators, and the new estimators. We note that for Clements' estimators, $\hat{C}_{pm}$ detects the shifts of A_1, A_2, and A_3. But, $\hat{C}_{pm}$ fails to differentiate the high-quality process A_2 (with 100 percent process yield) from the low-quality process A_3 (with only 50 percent process yield), as $\hat{C}_{pm} = 0.647$ for both A_2, and A_3. Therefore, we consider the estimator $\hat{C}_{pm}$ as inaccurate. For the modified estimators, $\hat{C}'_{pm}$, and $\hat{C}'_{pmk}$ detect the shifts of A_1, A_2, and A_3. But, $\hat{C}'_{pm}$ fails to differentiate the high-quality process A_2 (with 100 percent process yield) from the low-quality process A_3 (with only 50 percent process yield), as $\hat{C}'_{pm} = 0.647$ for both A_2, and A_3. Therefore, we consider the estimator $\hat{C}'_{pm}$ as inaccurate. On the other hand, the new estimators detect the shifts of A_1, A_2, and A_3. The new estimators also differentiate process A_2 from process A_3 (except for $\hat{C}''_p$ which never takes into account the process median and the target value, hence provides no sensitivity to process departure at all).

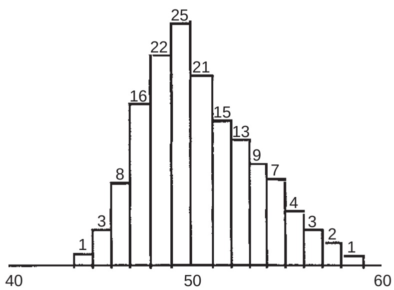

Figure 2(a)
The on-target process A with median $M = 50$, $U_p = 59$, and $L_p = 44$

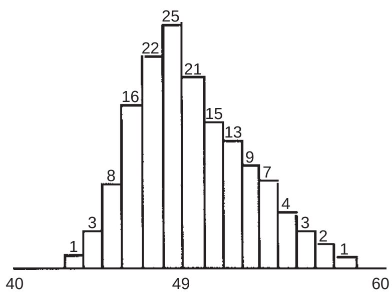

Figure 2(b)
The shifted process A_1 with median $M_1 = 49$, $U_p = 58$ and $L_p = 43$

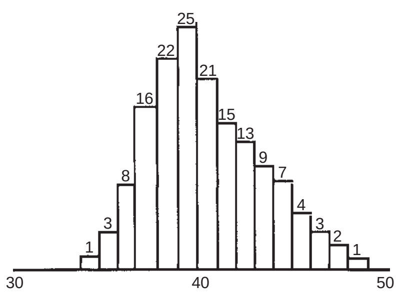

Figure 2(c)
The shifted process A_2 with median $M = 40$, $U_p = 49$ and $L_p = 34$

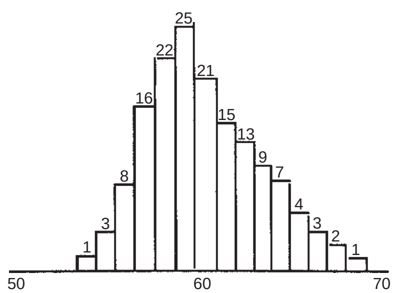

Figure 2(d)
The shifted process A_3 with median $M_3 = 60$, $U_p = 69$, and $L_p = 54$

Table V. Values of the estimators for processes A, $A_{1,2}$ and A_3 with $M = 50$, $M_1 = 49$, $M_2 = 40$ and $M_3 = 60$, where (LSL, T, USL) = (20, 50, 60)

	A	A_1	A_2	A_3
M	50	49	40	60
U_p	59	58	49	69
L_p	44	43	34	54
$\hat{C}_p$	2.667	2.667	2.667	2.667
$\hat{C}_{pk}$	1.111	1.222	2.222	0.000
$\hat{C}_{pm}$	2.666	2.476	0.647	0.647
$\hat{C}_{pmk}$	1.111	1.160	0.639	0.000
$\hat{C}'_p$	2.667	2.667	2.667	2.667
$\hat{C}'_{pk}$	1.333	1.467	2.666	0.000
$\hat{C}'_{pm}$	2.667	2.476	0.647	0.647
$\hat{C}'_{pmk}$	1.333	1.362	0.647	0.000
$\hat{C}''_p$	1.333	1.333	1.333	1.333
$\hat{C}''_{pk}$	1.333	1.289	0.889	0.000
$\hat{C}''_{pm}$	1.333	1.289	0.468	0.165
$\hat{C}''_{pmk}$	1.333	1.245	0.312	0.000

5. An application

To illustrate how the generalizations may be applied to the actual data collected from the factories, we present a case study on a MOSFET (metal-oxide-silicon field effect transistor) manufacturing process. The case which we studied was taken from an IC factory (located in Taiwan) which manufactures various types of semiconductor products. MOSFET is often applied on SRAM (static random access memory), DRAM (dynamic random access memory), and other IC products as an inverter or switch.

MOSFET has four terminals including:

(1) source (source of current);

(2) drain (destination of current);

(3) gate (switch);

(4) bulk (ground site).

An important function of the MOSFET is to control the current from the source terminal to the drain terminal. The threshold voltage V_t is one of the key parameters which determines the specifications of MOSFET. If the gate voltage is greater than the threshold voltage V_t, then an inversion layer is formed and the MOS channel (from the source to the drain) is turned on. On the other hand, if the gate voltage is smaller than the threshold voltage V_t, then no inversion layer is formed and the MOS channel is turned off. For the circuits to function properly, the threshold voltage V_t should be kept as low as possible to increase the transistor current driving capability. In the high-speed memory IC applications, the upper and lower specification limits, USL and LSL, for a particular model of MOSFET, the threshold voltages V_t are set to 0.5V and 0.7V respectively, where the target value T is set to 0.55V. The collected sample data (a total of 80 observations) are displayed in Table VI. This is a non-normal distribution (based on the 80 observations).

Figure 3 displays the histogram of the collected data. Figure 4 displays the corresponding box plot. Figure 5 displays the normal probability plot for the 80 observations. We perform Shapiro-Wilk test for normality check, obtaining $W = 0.93$ with p-value = 0.0002. We also perform Pearson's Chi-square test using the partition {0.553, 0.571, 0.589} obtaining Pearson $\chi^2 = 10$ with p-value = 0.0016. Since the p-values are sufficiently small for both tests, we may conclude that the data set comes from a non-normal distribution. To calculate the values of the estimators $\hat{C}''_p$, $\hat{C}''_{pk}$, $\hat{C}''_{pm}$ and $\hat{C}''_{pmk}$, we first proceed with calculating the following, and check Gruska *et al.* table (1989) to find U_p, L_p, and the sample median M obtaining:

Sample mean	0.571
Sample standard deviation	0.026
Sample skewness	0.662
Sample kurtosis	–0.252
Sample median M	0.576
99.865 percentile U_p	0.652
0.135 percentile L_p	0.534

We then calculate $d_u = \text{USL} - T = 0.7 - 0.58 = 0.12$, $d_l = T - \text{LSL} = 0.58 - 0.5 = 0.08$, $d = (\text{USL} - \text{LSL})/2 = (0.7 - 0.5)/2 = 0.1$, $d^* = \min\{d_u, d_l\} = 0.08$, and $a = \max\{d\,(M - T)/d_u,\ d\,(T - M)/d_l\} = \max\{0.1\,(0.576 - 0.580)/0.12,\ 0.1\,(0.580 - 0.576)/0.08\} = 0.005$. We also calculate $U_p - L_p = 0.118$, $(U_p - L_p)\,6 = 0.02$, $d^*/d_u = 0.667$, and $d^*/d_l = 1.00$. Substituting these values into the definitions of the

0.53	0.53	0.53	0.54	0.54	0.54	0.54	0.54	0.54	0.54
0.54	0.54	0.54	0.55	0.55	0.55	0.55	0.55	0.55	0.55
0.55	0.55	0.55	0.55	0.55	0.55	0.56	0.56	0.56	0.56
0.56	0.56	0.56	0.56	0.56	0.56	0.56	0.56	0.56	0.56
0.57	0.57	0.57	0.57	0.57	0.57	0.57	0.57	0.57	0.57
0.58	0.58	0.58	0.58	0.58	0.58	0.58	0.58	0.58	0.58
0.59	0.59	0.59	0.59	0.59	0.60	0.60	0.60	0.60	0.60
0.61	0.61	0.61	0.61	0.62	0.62	0.62	0.62	0.62	0.62

Table VI. Values of V_t (threshold voltage) of MOSFET

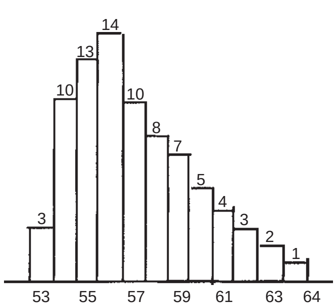

Figure 3. Histogram of the collected data

new estimators, we obtain

$$\hat{C}_p'' = \frac{2 \times 0.1}{0.118} = 1.695$$

$$\hat{C}_{pk}'' = \min \left\{ \frac{0.7 - 0.576}{0.059} \times 0.667, \frac{0.576 - 0.05}{0.059} \times 1 \right\} = 1.288$$

$$\hat{C}_{pm}'' = \frac{2 \times 0.08}{6\sqrt{0.02^2 + 0.005^2}} = 1.294$$

$$\hat{C}_{pmk}'' = \min \left\{ \frac{0.7 - 0.576}{3\sqrt{0.02^2 + 0.005^2}} \times 0.667, \frac{0.576 - 0.5}{3\sqrt{0.02^2 + 0.005^2}} \right\} \times 1 = 1.229$$

We also calculate the four index values using the original Clements' and the modified Clements' methods, obtaining $\hat{C}_p$ = 1.695, $\hat{C}_{pk}$ = 1.632, $\hat{C}_{pm}$ = 1.646, $\hat{C}_{pmk}$ = 1.609, the modified Clements' estimators, $\hat{C}_p'$ = 1.695, $\hat{C}_{pk}'$ = 1.288, $\hat{C}_{pm}'$ = 1.646, $\hat{C}_{pmk}'$ = 1.251. We note that all four index values are greater than 1.00. Thus, we conclude that the process is capable (adequate with respect to the given manufacturing specifications). In fact, there are zero observations falling outside the specification interval (LSL, USL).

In this example, the process variation is small relative to the specification tolerance ($\hat{C}_p''$ = 1.695). The process departure is also relatively insignificant (max {(0.576 − 0.580)/0.12, (0.580 − 0.576)/0.08} = 0.05). Therefore, the four index values calculated using three different methods are not significantly different from each other. But, if a process shift occurs, then only the proposed generalization can detect such changes. In this case, the index values calculated using the proposed generalization would significantly decrease.

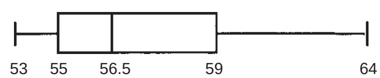

Figure 4.
The box plot of the collected data

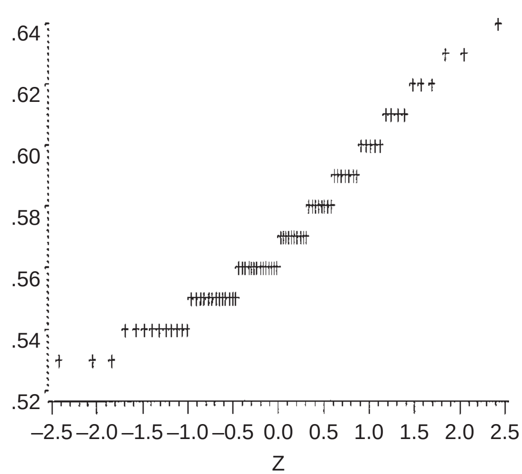

Figure 5.
The normal probability plot of the collected data (80 observations)

6. Conclusion

For non-normal Pearsonian processes, Clements (1989) proposed a method for calculating estimators of the two basic process capability indices C_p and C_{pk}. Pearn and Kotz (1994) applied Clements' method to obtain estimators for the other two more advanced process capability indices C_{pm} and C_{pmk}. Unfortunately, their investigation was restricted to processes with symmetric tolerances. In this paper, we considered a generalization of Clements' method to handle cases with asymmetric tolerances. The generalization takes into account the asymmetry of the tolerances. Comparisons between the generalization and the original Clements' method are provided. The results showed that the proposed generalization is more sensitive than the original Clements' and the modified Clements' methods in detecting process shift. The results also showed that the proposed generalization is more accurate than the original Clements' and the modified Clements' methods in measuring process capability.

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